

Design Of Passive Hydro-Pneumatic Suspension System For Car

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Abstract

The abstract Suspension system is most important in a vehicle which counters the disturbances generated from the road. It contributes significantly on the vehicle's stability, safety and control. There are different types of suspension systems used depending on the type of vehicle, road conditions and speed of movement. The different forms of suspensions include mechanical, pneumatic, hydro-pneumatic and electromechanical each with active, semi – active or passive type. The hydro-pneumatic system uses an accumulator to generate spring force and a remote valve block to generate damping force. A hydraulic cylinder replaces the damper strut and springs of the vehicle. The cylinder generates oil volume displacement towards the accumulator. The oil is assumed to be incompressible and the volume of the air chamber inside the accumulator is diminished which creates a pressure increase by means of the “ideal gas law”. Higher pressure results in a higher reaction force and so a spring is established. The flows from the piston chambers and rod chambers of the cylinders are led through the tubing system and a flow resistor. Due to the pressure losses energy is dissipated and damping is generated. This paper is an over view on design of hydro-pneumatic suspension system. A quarter cars approach with a passive Hydro-pneumatic suspension have been presented in this paper in which static and dynamic load is calculated considering all arbitrary conditions to design the system. The initial pressure and volume of an accumulator and dimensions of a single acting cylinder without preload is derived.

Keywords: Accumulator, single acting cylinder, Hydro-pneumatic suspension, flow resistor, static and dynamic load, initial pressure.

1. Introduction

The main components of Hydro-pneumatic suspension system are accumulators, cylinders, flow resistors, lines and fittings.

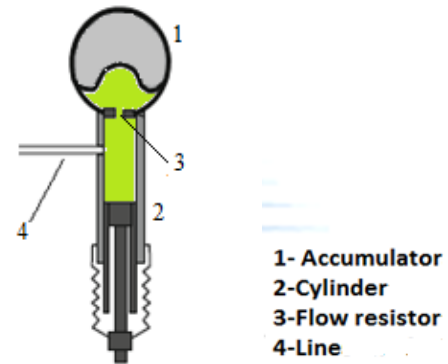


Fig.1.1 Elements of hydro-pneumatic suspension

The basic layout of the hydro-pneumatic suspension system is shown in the Figure 1.1. The Cylinders are load carrying elements in the system, transfers the forces between the input side and isolated side. It also provides travel of the suspension. The Accumulators are the elements which provide the spring function through an elastic medium. Generally the accumulators are preloaded with gas, as gas is compressible and it provides increasing pressures with increasing loads. The Flow Resistors (orifice) are important elements for the system as they provide the required damping to absorb the disturbances. The Hydraulic lines and fittings are the elements which transfer mechanical power from one place to other of the circuit in form of hydraulic/ pneumatic energy. They also act as controlling devices [3].

2. Importance of Hydro-Pneumatic System

2.1. Spring characteristics

The Hydro-pneumatic system is developed for better comfort and handling while driving over uneven roads which require no additional damping system. The damping in this system is formed by friction in the suspension joints and in the hydraulic cylinder and pressure loss created over the tubing system. The damping coefficient can be freely selected by the driver as well as the ride height and stiffness can be varied also. Hydro-pneumatic Suspension system has progressive spring-rate and spring-rate is completely adaptive to the load acting on it. The stiffness can be varied from soft to very hard all by its own. The continuous Self leveling system allows the passenger his safety and comfort which is an added advantage. The wheel travel is fixed no matter what load is acting on it [2].

2.2. Damping characteristics

The damping coefficient can be freely selected by the driver as well as the ride height and stiffness can be varied also. The characteristic equation for the system is,

$$mx^2 + c_s + k = 0$$

This equation determines the two independent roots for the damped vibration problem. The roots to the characteristic equation fall into one of the following 3 cases:

If $[c_s^2 - 4mk < 0]$ the system is termed under-damped. The roots of the characteristic equation are complex conjugates, corresponding to oscillatory motion with an exponential decay in amplitude.

If $[c_s^2 - 4mk = 0]$ the system is termed critically-damped. The roots of the characteristic equation are repeated, corresponding to simple decaying motion with at most one overshoot of the system's resting position.

If $[c_s^2 - 4mk > 0]$ the system is termed over-damped. The roots of the characteristic equation are purely real and distinct, corresponding to simple exponentially decaying motion [4].

3. Operating Principle

The hydraulic pressure should be adjusted for the static loads to a required level by adding or releasing the hydraulic fluid from the accumulator. As the piston moves towards the piston side due to the static load, the fluid volume in the accumulator changes and hence the pressure also changes. Gas, which is the other fluid in the accumulator, gets compressed and exerts a force on the piston rod. This defines the spring rate of the system. The force acting on piston is always equal to the forces resulting from the pressures acting. When the force is increased due to the road profile, and the piston is displaced by a distance 'x', the hydraulic fluid is displaced into the accumulator which changes the pressure. This change proceeds until the pressure in the accumulator has reached a certain level which again provides a balance for the system.

The damping effect can be calculated using Stiffness as a parameter given by [2];

$$F_s = A_p \times P_s$$

$$F_d = A_p \times P_{sys}$$

$$K = \frac{F_s - F_d}{\Delta x}$$

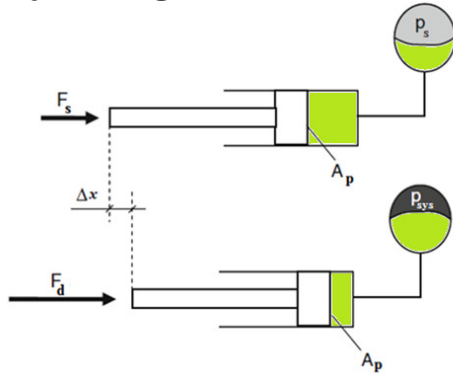


Fig.3.1.Operating principle of hydro-pneumatic suspension

4. Dynamic Load Calculations

4.1. Specifications of car selected

The basic specifications of the car selected (HONDA CIVIC) are collected and is represented in the table [5]. These values are applied to calculate some of the basic loads [1].

Table 4.1: Car specifications

Wheel Base (L)	2621 mm
Front Track-width (T _r)	1471 mm
Rear Track-width (T _r)	1468 mm
Total Kerb Weight (W)	1350 Kg
Fore length (b)	1038 mm
Aft length (c)	1583 mm
Height of CG (h)	513 mm
0 -100 kmph	11.2 sec
Wheels	Alloy wheels, 195/ 65 R15

4.2. Assumed parameters

- Front suspension: :McPherson suspension
- Tires: :195/65 R15 tubeless tires
- Speed of the vehicle (V) : 100 Kmph
- Weight of car including driver : 1600 Kg
- Turning Radius (R) : 80 m

- Front Slip angle(α_f) : 4.010 [Slalom test@40 Kmph]

4.3. Calculations of forces acting on front individual wheel

4.3.1. Static load (W_z)

The static weight acting on front individual wheel is calculated using the formula;

$$\begin{aligned}
 W_z &= \frac{W \times c}{2L} \times \cos \theta \\
 &= \frac{(1600 \times 1583)}{(2 \times 2621)} \times \cos 0 = 483.174 \text{ kg} \\
 &= 483.174 \times 9.81 \\
 &= 4.74 \text{ KN} \quad (\text{Maximum static load on one wheel})
 \end{aligned}$$

4.3.2. Load transfer due to Acceleration (W_x)

Calculation of linear acceleration 'a_x':-

From the table we can see that 0-100 kmph (27.78 m/s) in 11.2 sec. So, using Newton's laws of motion, the acceleration is;

$$\begin{aligned}
 V &= U + a \times t \\
 27.78 &= 0 + a_x \times 11.2 \\
 a_x &= 2.48 \text{ m/s}^2
 \end{aligned}$$

$$\text{So; } W_x = \frac{W \times a_x \times h}{2 \times g \times l}$$

$$\begin{aligned}
 W_x &= \frac{1600 \times 2.48 \times 513}{2 \times 9.81 \times 2621} \\
 &= 39.58 \text{ Kg} \\
 &= 39.58 \times 9.81 \\
 &= 0.388 \text{ KN}
 \end{aligned}$$

The load transfer due to acceleration is on the rear axle, so this load is deducted from front axle load.

$$W_x = -0.388 \text{ KN}$$

4.3.3. Lateral weight transfer (W_y)

The lateral force depends on the front axle load and centrifugal force acting on the vehicle. The lateral acceleration can be calculated using the formula;

$$a_y = \frac{v^2}{R}$$

$$= \frac{(27.78)^2}{80}$$

$$= 9.646 \text{ m/s}^2$$

So;

$$W_y = \frac{W \times a_y \times c}{g \times l}$$

$$= \frac{1600 \times 27.78^2 \times 1583}{9.81 \times 80 \times 2621}$$

$$= 950.25 \text{ kg} \times 9.81$$

$$= 9.322 \text{ KN}$$

4.3.4. Braking force (F_b)

Vehicle decelerates (i.e. braking) at a constant 0.5 × g

$$D_x = 0.5 \times g$$

$$F_b = \mu \times \frac{W \times D_x \times h}{2 \times g \times l}$$

Coefficients of friction of tires are about μ = 0.7 for dry roads and μ = 0.4 for wet roads.

$$F_b = 0.7 \times \frac{1600 \times 0.5 \times g \times 513}{2 \times g \times 2621}$$

$$= 54.8 \text{ kg}$$

$$= 54.8 \times 9.81$$

$$= 0.5376 \text{ KN}$$

4.3.5. Force due to road hump(F_h)

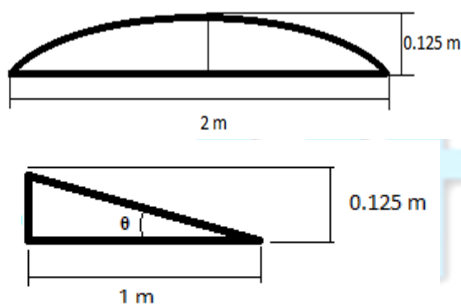


Fig.4.1. Road hump

Dimensions of road hump are as per Indian design standards and vehicle speed on humps should be not more than 25 – 30 Kmph

Let car speed U = 30 Kmph = 8.33 m/s

So, to cover 1 m time $t = \frac{1}{8.33} = 0.12 \text{ sec}$

$$\theta = \tan^{-1}\left(\frac{0.125}{1}\right) = 7.125^\circ$$

$$U_{\text{horizontal}} = U \cos \theta = 8.33 \cos(7.125) = 8.266 \text{ m/s}$$

$$U_{\text{vertical}} = U \sin \theta = 8.33 \sin(7.125) = 1.033 \text{ m/s}$$

$$\frac{dU_{\text{vertical}}}{dt} = \frac{0 - 1.033}{0 - 0.12} = 8.61 \text{ m/s}^2$$

$$F_h = \text{mass} \times \text{acceleration} = 483.174 \times 8.61 = 4.16 \text{ KN}$$

4.3.6. Load transfer due to downhill on front wheel

$$W_{\text{dh}} = \frac{w \times h}{2 \times l} \sin \theta$$

The influence of grade on axle loads is also worth considering. Grade is defined as the "rise" over the "run." That ratio is the tangent of the grade angle, El. The common grades on interstate highways are limited to 4% wherever possible. On primary and secondary roads they occasionally reach 10 to 12%.

Table 4.2: The slope of a road

Angle	% rise / run	% rise / hyp	%Diff
5	8.74887	8.71557	100.3820
10	17.63270	17.36482	101.5427
15	26.79492	25.88190	103.5276
20	36.39702	34.20201	106.4178
25	46.63077	42.26183	110.3378
30	57.73503	50.00000	115.4701
35	70.02075	57.35764	122.0775
40	83.90996	64.27876	130.5407
45	100.00000	70.71068	141.4214
50	119.17536	76.60444	155.5724
55	142.81480	81.91520	174.3447
60	173.20508	86.60254	200.0000
65	214.45069	90.63078	236.6202
70	274.74774	93.96926	292.3804
75	373.20508	96.59258	386.3703
80	567.12818	98.48078	575.8770
85	1,143.00523	99.61947	1,147.3713
90	Infinite	100.00000	Infinite

From above table grade of 12% the angle $\theta = 10^\circ$ is considered [7];

$$W_{dh} = \frac{1600 \times 513}{2 \times 2621} \sin 10 = 27.19 \text{ kg}$$

$$= 27.19 \times 9.81$$

$$= 0.267 \text{ KN}$$

Total Load

Now, the total Dynamic load acting on a vehicle is calculated by [1];

$$F_d = \frac{W \times c}{2L} \times \cos \theta - \frac{W \times a_x \times h}{2 \times g \times l} + \frac{W \times a_y \times c}{g \times l} + \mu \times \frac{W \times D_x \times h}{2 \times g \times l} + F_h + \frac{w \times h}{2 \times l} \sin \theta$$

$$F_d = 4.740 - 0.388 + 9.322 + 0.5376 + 4.16 + 0.267$$

$$F_d = 18.64 \text{ KN}$$

Note: All conditions discussed above will not be present at same time so, dynamic load value is not fixed. The total load on one front wheel is calculated considering worst case [1].

5. Dimensioning of The Hydro-Pneumatic Suspension Hardware

5.1. Cylinder

The dimension of the cylinder piston diameter is important as it ensures the maximum utilization of the system pressure. As per Standard sizes of piston seal available; seal of 63 mm (So, the diameter of piston $d_p = 0.063 \text{ m}$) diameter is selected, so system pressure P_{sys} can be calculated as [2];

$$P_{sys} = \frac{F_d}{A_p} = \frac{F_d}{\frac{\pi}{4} d_p^2}$$

$$= \frac{4 \times 18.64 \times 10^3}{3.14 \times 0.063^2} = 59.83 \text{ bar} = 5.983 \text{ MPa}$$

5.2. Piston rod diameter (d_r)

Material used for manufacturing: Mild steel UNS G10180

(Density 7.87 g/cm³; Tensile strength 440 MPa; Yield strength 370 MPa)

Yield strength = 370 MPa; Consider FOS = 6

$$6 = \frac{370 \text{ MPa}}{\text{Design stress}}$$

Designed stress = 61.67 MPa

$$d_r = \sqrt{\frac{4 \times F_d}{\pi \times P_{sys}}}$$

$$= \sqrt{\frac{4 \times 18.64 \times 10^3}{3.14 \times 61.67 \times 10^6}}$$

$$= 0.0196 \text{ m} = 20 \text{ mm}$$

- Diameter of the piston = 63 mm
- Piston rod diameter = 20 mm
- Area of the piston = $\frac{\pi}{4} d_p^2 = \frac{\pi}{4} \times 0.063^2$
= $3.117 \times 10^{-3} \text{ m}^2$
- Cylinder inner Diameter = 63 mm
- Cylinder length = 200 mm
- Stroke length = 130 mm

5.3. Cylinder thickness

Material used for manufacturing: Mild steel **UNS G10180** (Density 7.87 g/cm³; Tensile strength 440 MPa; Yield strength 370 MPa)

P_{sys} = Cylinder inner pressure = 59.83 bar = 5.983 MPa

P_o = cylinder outer pressure = 1 atm = 0.101325 bar

r_i = cylinder inner radius = 31.5 mm,

Applying maximum principal stress theory of failure;

Yield strength = 370 MPa; Consider FOS = 6

$$6 = \frac{370 \text{ MPa}}{\text{Design stress}}$$

Design stress = 61.67 MPa

The Hoop stress is always maximum than axial and radial stresses. Considering Hoop stress = 61.67 MPa, then outer diameter is found by [6];

$$\sigma_c = [(p_i r_i^2 - p_o r_o^2) / (r_o^2 - r_i^2)] - [r_i^2 r_o^2 (p_o - p_i) / (r^2 (r_o^2 - r_i^2))]$$

Where

- σ_c = stress in circumferential direction (MPa)
- r = radius to point in tube or cylinder wall (mm)
- ($r_i < r < r_o$)
- Maximum stress when r = r_i (inside pipe or cylinder)

$$61.67 = \left[\frac{(5.983 * 0.0315^2 - 0.101325 * r_o^2)}{(r_o^2 - 0.0315^2)} \right] -$$

$$\left[\frac{0.0315^2 * r_o^2 (0.101325 - 5.983)}{0.0315^2 (r_o^2 - 0.0315^2)} \right]$$

$$r_o^2 = \frac{(5.983 * 0.0315^2 + 61.67 * 0.0315^2)}{(61.67 + 0.101325 - 5.983)}$$

$$r_o = 34.69 \text{ mm}$$

$$t = 34.69 - 31.5 = 3.19 \text{ mm}$$

Instead of thickness t = 3.19 mm, taking thickness t = 3.5 mm and checked principal stresses;

So, r_o = cylinder outer radius = 35 mm

5.3.1. Stress in axial direction

The stress in axial direction at a point in the tube or cylinder wall can be expressed as [6];

$$\sigma_a = (p_i r_i^2 - p_o r_o^2) / (r_o^2 - r_i^2)$$

Where;

- σ_a = stress in axial direction (MPa)
- p_i = internal pressure in the tube or cylinder (MPa)
- p_o = external pressure in the tube or cylinder (MPa)
- r_i = internal radius of tube or cylinder (mm)
- r_o = external radius of tube or cylinder (mm)

$$\sigma_a = (5.983 * 0.0315^2 - 0.101325 * 0.035^2) / (0.035^2 - 0.0315^2) = 25 \text{ MPa}$$

5.3.2. Stress in circumferential direction - hoop stress

The stress in circumferential direction - hoop stress - at a point in the tube or cylinder wall can be expressed as [6];

$$\sigma_c = [(p_i r_i^2 - p_o r_o^2) / (r_o^2 - r_i^2)] - [r_i^2 r_o^2 (p_o - p_i) / (r^2 (r_o^2 - r_i^2))]$$

Where

- σ_c = stress in circumferential direction (MPa)
- r = radius to point in tube or cylinder wall (mm) ($r_i < r < r_o$)
- Maximum stress when r = r_i (inside pipe or cylinder)

$$\sigma_c = \left[\frac{(5.983 * 0.0315^2 - 0.101325 * 0.035^2)}{(0.035^2 - 0.0315^2)} \right] -$$

$$\left[\frac{0.0315^2 * 0.035^2 (0.101325 - 5.983)}{(0.0315^2 (0.035^2 - 0.0315^2))} \right]$$

$$= 55.9 \text{ MPa}$$

5.3.3. Stress in radial direction

The stress in radial direction at a point in the tube or cylinder wall can be expressed as [6];

$$\sigma_r = [(p_i r_i^2 - p_o r_o^2) / (r_o^2 - r_i^2)] + [r_i^2 r_o^2 (p_o - p_i) / (r^2 (r_o^2 - r_i^2))]$$

Maximum stress when r = r_o (outside pipe or cylinder)

$$\sigma_r = \left[\frac{(5.983 * 0.0315^2 - 0.101325 * 0.035^2)}{(0.035^2 - 0.0315^2)} \right] +$$

$$\left[\frac{0.0315^2 * 0.035^2 (0.101325 - 5.983)}{(0.0315^2 (0.035^2 - 0.0315^2))} \right] = - 5.983 \text{ MPa}$$

According to maximum principal stress theory of failure;

$$FOS = \frac{\text{Material yield strength}}{\text{Maximum principal stress}} = \frac{370 \text{ MPa}}{55.9 \text{ MPa}} = 6.6$$

So, cylinder thickness t = 3.5 mm

5.4. Accumulator

$$P_{\min} = F_s / A_p = 4.74 \text{ KN} \times 3.117 \times 10^{-3} \text{ m}^3 = 14.78 \text{ bar}$$

$$P_{\max} = 59.83 \text{ bar}$$

Maximum volume of oil displaced in cylinder = $A_p \times$

Stroke length = $3.117 \times 10^{-3} \times 0.130$

$$= 4.0524 \times 10^{-4} \text{ m}^3 = 0.405 \text{ liter}$$

From this value we can select accumulator close to required value is of 0.5 liter from the standard capacities available [8].

Table 5.1: Technical details about accumulator selected

Designation	ELM 0.50-210/00/CF
Execution form	CF
Volume V_0 in liter	0.50
Max pressure in bar	210
Max pre-pressure in bar	130
Compression ratio P_{max}/P_0	8
Pressure Amplitude P_{max}/P_{min}	175
Weight in kg	2
A	163
B	33
SW	41
D	106
G	-
F	G 1/2
H	M33x1.5
Clamp designation	E106
Lock-nut designation	M33

The above measurements are given in mm and do not take manufacturing tolerance into consideration [8].

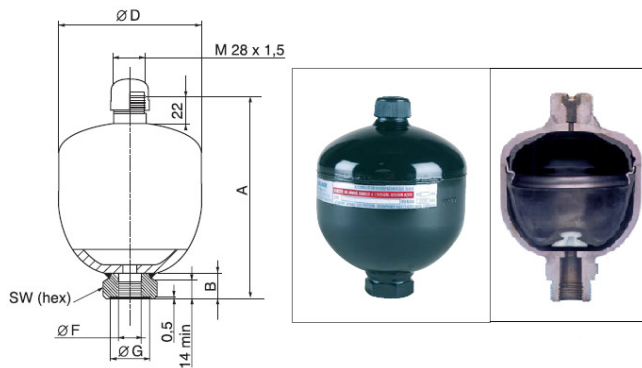


Fig.4.2. Diaphragm type accumulator

6. Detailed Calculation of P_0 and V_0

This pressure range is defined by two basic criteria:

1. Maximum pressure criterion: the design, in particular the material and the dimensioning of the outer shell, defines the permissible maximum pressure for an accumulator. It must not be exceeded in any operating condition if the accumulator has to be fatigue resistant throughout the lifetime of the suspension [2].
2. Diaphragm deformation criterion: the diaphragm has a permissible maximum deformation for its deflection inside the accumulator. This deformation must not be exceeded during the oscillation of the diaphragm position throughout the entire operating time of the suspension system. A simple guideline for the use of diaphragm accumulators in hydro-pneumatic suspensions is the 10% rule. It states that at any time during operation, the inner accumulator volume should be filled with either at least 10% hydraulic fluid or with 10% gas to ensure that the maximum deformation is not exceeded [2].

In the following, isothermal changes of state are used for the calculations since this represents the more critical condition concerning the inner volume portion of oil and gas and their minimum values. An accumulator with a pre-charge pressure p_0 and a volume V_0 must be operated (according to the diaphragm deformation criterion) between $V = 0.1 \cdot V_0$ and $V = 0.9 \cdot V_0$. For the isothermal change of state, the minimum and maximum pressure can be calculated [2];

$$P_{min} = \frac{p_0 V_0}{0.9 V_0} = 1.11 p_0$$

$$P_{max} = \frac{p_0 V_0}{0.1 V_0} = 10 p_0$$

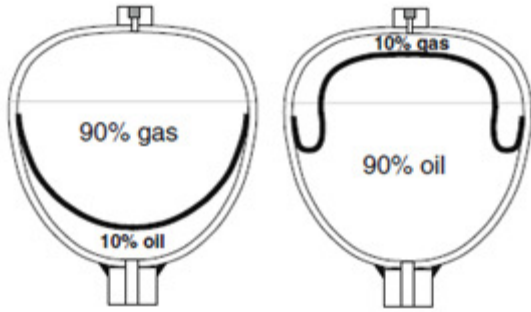


Fig.4.2. Limits for diaphragm deformation for the application in a suspension system

The permissible pressure ratio p_{max}/p_{min} is therefore 9. For a temperature range from -20 to $+60^{\circ}C$ the pressure range is [2];

$$p_{min} = 1.11p_0 \frac{333.15K}{293.15K} = 1.26p_0$$

$$p_{max} = 10p_0 \frac{253.15K}{293.15K} = 8.64p_0$$

This looks like a minor change but actually the permissible pressure ratio has been reduced from 9 to the value of 6.85

There is another parameter influencing the pressure ratio: the production process always contains small errors, also with regards to the gas pre-charge pressure. Therefore the pre-charge pressure will not always have the exact value but will have a slight production tolerance. For later calculations a tolerance of $\pm 5\%$ is assumed and the pressure limits will be even more constricted. A positive deviation of the pre-charge pressure will increase the minimum pressure; a negative deviation decreases the maximum pressure [2];

$$p_{min} = 1.26p_0 \times 1.05 = 1.32p_0$$

$$p_{max} = 8.64p_0 \times 0.95 = 8.21p_0$$

This additionally lowers the pressure ratio to 6.2.

Another peculiarity of diaphragm accumulators must be considered for the correct choice of parameters. The gas diffuses through the diaphragm into the hydraulic fluid and therefore is partially lost – and the suspension

characteristics with it. So the accumulator is subjected to a diffusion pressure loss. Although this is not a problem for the minimum pressure, it has to be considered when calculating the maximum pressure. Assuming a permissible pressure loss of 10% of the pre-charge pressure between service (and refill) intervals, the maximum pressure is reduced to [2];

$$p_{max} = 8.21p_0 \times 0.9 = 7.39p_0$$

The permissible pressure ratio then is 5.6. So the formerly good pressure ratio of 9 has shrunk to a value of 5.6. So, we can conclude conceptually good pressure ratio is 5.6

$$p_{min} = 1.26p_0 \times 1.05 = 1.32p_0$$

$$P_0 = \frac{1}{1.32} \times 14.78 \text{ bar} = 11.19 \text{ bar}$$

$$p_{max} = 8.21p_0 \times 0.9 = 7.39p_0$$

$$P_{max} = 7.39 P_0 = 7.39 \times 11.19 \text{ bar}$$

$$= 82.69 \text{ bar} \quad (\text{Considering all factors})$$

$$\text{Good pressure ratio} = \frac{82.69 \text{ bar}}{14.78 \text{ bar}} = 5.6$$

(Considering all factors)

As, $P_{min} = 14.78 \text{ bar}$ & $P_{max} = 59.83 \text{ bar}$

The design pressure ratio is;

$$\frac{P_{max}}{P_{min}} = \frac{59.83 \text{ bar}}{14.78 \text{ bar}} = 3.69 = 4.048 \quad (\text{Safe side})$$

4. Conclusions

System is designed as per the general design procedure that is first need definition as importance of hydro-pneumatic suspension system explained above. The total load acting on the system is calculated according to vehicle dynamics considering all arbitrary loads acting on individual front wheel of car. For that load piston and cylinder assembly is designed using maximum principle

stress theory of failure. According to maximum oil volume displaced the capacity of diaphragm type accumulator is selected from olaer catalogue. Detailed calculation of initial (pre-charge) pressure and volume is done and good pressure ratio is found 5.6. Minimum and maximum pressure values calculated to find design pressure ratio which is 4. The next step will be development of system and its performance evaluation.

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